Influence of Loading Behavior on the Post Buckling of Circular Rings

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IN the following we are concerned with the initial post buckling behavior[1] of a circular ring under 3 different types of conservative external pressure: the constant direction, the radially directed, and the hydrostatic pressure. The bending energy of a circular ring is

$$\pi_B = \oint \int_0^{\dot\psi} M \mathrm{d}\dot\psi \; \mathrm{d}\varphi \tag{1}$$

where the bending moment is

$$M = E\int_F z\mathrm{d}/F(1+z/r)\overset{\circ}{\epsilon} + E\int_F z^2\mathrm{d}/F(1+z/r)\,\dot\psi$$

F is the sectional area, E is the modulus of elasticity, z is the ring space coordinate, r is the ring radius,

$$\psi = \sin^{-1}\left[\frac{v+\dot w}{r(1+\overset{\circ}{\epsilon})}\right]$$

is the angle of rotation, w is radial, v is the tangential displacement component, $(\dot{\ }) = \mathrm{d}(\ )/\mathrm{d}\varphi$, φ is the angular coordinate and

$$\overset{\circ}{\epsilon} = \left[(1+\frac{w}{r} - \frac{\dot v}{r})^2 + (\frac{v}{r} + \frac{\dot w}{r})^2\right]^{1/2} - 1$$

is the axial strain. Assuming axial inextensibility ($\overset{\circ}{\epsilon} = 0$) and neglecting z/r compared with unity, the total internal energy is equal to the bending energy which follows from Eq. (1) as

$$\pi_i = (EI/2r^2)\oint[(1/r)\ (\dot v + \ddot w)^2 + (1/2r^3)\ (\dot v + \ddot w)^2\ (v+\dot w)^2 + \ldots]\mathrm{d}\varphi \tag{2}$$

where I is the moment of inertia. The loading energy of the constant directional (dead) pressure P, where the load vector is supposed to maintain its direction and absolute value, is

$$\pi_a = P\oint wr\mathrm{d}\varphi \tag{3}$$

The loading energy of the centrally directed pressure where the load vector is supposed to rotate to remain directed towards the center is

$$\pi_b = P\oint \Delta r r\mathrm{d}\varphi \tag{4}$$

Here Δr is the changement of the radius occurring after buckling. Thus, we have up to second-order terms exact, the following external potential

$$\pi_b = \pi_a + P\oint v^2\mathrm{d}\varphi \tag{5}$$

Finally, the loading energy of the hydrostatic pressure, which is equal to the product of the external pressure and the difference of the area of the buckled and unbuckled configuration, can be written as

$$\pi_c = \pi_b + P\oint (w^2 - w\dot v + v\dot w)\ \mathrm{d}\varphi \tag{6}$$

Considering the auxillary condition (due to $\epsilon = 0$)

$$\frac{1}{r}\ (w-\dot v) + \frac{1}{2r^2}\ [(w-\dot v)^2 + (\dot w + v)^2] = 0 \tag{7}$$

in eliminating w from Eq. (3), the following components of the potential functional $W = \Sigma\ \pi = W_2 + W_3 + \ldots$ are obtained

$$W_{2a} = \frac{1}{2}\frac{EI}{r^3}\oint(\dot v + \ddot w)^2\mathrm{d}y - \frac{P}{2}\oint[(w-\dot v)^2 + (\dot w + v)^2]\ \mathrm{d}\varphi \tag{8}$$

$$W_{2b} = W_{2a} + \oint \frac{P}{2}\ v^2\mathrm{d}\varphi \tag{9}$$

$$W_{2c} = W_{2b} + \oint \frac{P}{2}\ (w^2 + \dot w v - w\dot v)\ \mathrm{d}\varphi \tag{10}$$

$$W_3 = 0 \tag{11}$$

$$W_4 = \frac{1}{2}\frac{EI}{r^5}\oint(\dot v + \ddot w)^2(v+\dot w)^2]\ \mathrm{d}\varphi \tag{12}$$

Since the potential functional is totally symmetric, the axial inextensibility condition of the branching load will hold true for the initial curvature of the post buckling path and it is easy to show that

$$w = \dot v \tag{13}$$

$$w = a\cos 2\varphi \tag{14}$$

and

$$v = (1/2)\ a\sin 2\varphi \tag{15}$$

is the exact solution of the problem. Omitting the details of the elementary perturbation procedure, the results are represented in Table 1. We also might mention that for the

Table 1 Initial post buckling of the circular ring under various loading and supporting conditions ($a = w_{max}$)

Loading condition	Initial post buckling path	
	Ring sliding freely on a single fixed axis[5,6]	Ring sliding freely on a fixed cross[5,6]
Constant directional pressure	$Pr^3/EI = 3.27 + (27/8)0.89(a/r)^2$	$Pr^3/EI = 4.0 + (27/8)1.33(a/r)^2$
Centrally directed pressure	$= 4.5 + (27/8)1.69(a/r)^2$	$= 4.5 + (27/8)1.69(a/r)^2$
Hydrostatic	$= 3.0 + (27/8)0.75(a/r)^2$	$= 3.0 + (27/8)0.75(a/r)^2$

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case of the constant directional pressure, two different answers exist. This is so because the fundamental path, in this particular case, is unstable.[5,6] The lower value of 3.27 is an upper bound and corresponds to the case of the nonvanishing mean rotation. In this case, the ring is allowed to slide freely on a single fixed axis while the higher value of 4 corresponds to the case when the ring is allowed to slide freely on a fixed cross.[5,6] The classical buckling load of all 3 cases otherwise agrees with the well-known ones.[2]

Finally, we might mention that the only initial post buckling behavior study of the ring under external pressure known to the Author is that of Ref. 3 for the cases of hydrostatic and constant directional pressure. However, the conclusion drawn there that the ring is imperfection sensitive[1] is erroneous due to inadequate formulations. The stability performance of the ring is dominated by bending and not membrane energy. This is the case with all structures with geometry and boundary conditions which allows quasi-inextensional deformation. To obtain the expected rise in the post buckling path, we have to consider nonlinear terms in the expression for the rotation. However, such terms are neglected in the expression for rotation used in Ref. 3. The Sanders' equations are appropriate only for problems in which the middle-surface membrane energy is dominating. Also, the two possible ways of holding the ring in space were omitted in Ref. 3. A similar result for the ring in an elastic foundation was given in Ref. 4. Thus. the ring possesses a symmetric stable point of bifurcation for a variety of loading and supporting conditions.

References

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Effect of Neutron Radiation on the Vaporization of Ammonium Perchlorate

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Nomenclature

α = fraction of ammonium perchlorate vaporized
k = rate coefficient for AP vaporization
t = time
$f(\alpha)$ = $1-(1-\alpha)^{1/2}$

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Table 1 Rate coefficients for the vaporization of a neutron-irradiated and non-irradiated ammonium perchlorate[a]

Run	T,K	mass,mg	$k\times10^4$, sec^{-1} [b]
1	592	11.8	2.8
2	592	12.1	3.2
3	592	12.7	2.8
4	592	12.7	3.0
5[c]	574	13.6	0.49
6	576	13.1	0.53
7[c]	576	6.10	1.5
8	576	6.04	1.7
9[c]	576	6.00	1.7
10	576	5.92	1.7
11[c]	576	6.06	1.7
12	576	6.06	1.4
13[c]	577	6.08	1.6
14	577	6.10	1.9

[a]0.1 1/min flow rate of argon.
[b]$1-(1-\alpha)^{1/2}=kt$.
[c]Neutron-Irradiated AP 4.2×10^{14} neutrons/cm^2.

Introduction

IT has been well established that X-ray, γ-ray, and neutron radiation alter the induction period and low-temperature decomposition of ammonium perchlorate (AP).[1-7] These changes indicate that such radiation causes both physical and chemical changes in AP.[8] Such results shed no light on the effect of radiation on the combustion rate of AP or AP-based composite propellants. This is because a number of workers have repeatedly shown that it is possible to increase or decrease the rate of the low-temperature reaction and not affect the combustion rate of composite propellants made from the altered AP.[9-13]

To gain insight into the effect of ionizing radiation on composite-propellant combustion, we examined the effect of neutron radiation on the kinetics of vaporization of AP by isothermal thermogravimetry. The vaporization of AP is deemed the only condensed phase reaction relevant to the combustion of AP.[14] The neutron radiation will interact most strongly with the condensed phase. This knowledge of the effect of neutron radiation on the rate of vaporization of AP will enable us to infer the effect of neutron radiation on the combustion rate of AP, and AP composite propellants. To date no one has examined the effect of any form of ionizing radiation on the kinetics of AP vaporization.

Experimental

Finely-ground AP from Hercules Alleghany Ballistics Laboratory, Cumberland, Md., was used for these experiments. This is the same form of AP in SPRINT propellant. The AP was dried, and one-half of the sample subjected to neutron radiation at the Army Pulsed Radiation Facility, Aberdeen Proving Ground, Md. Sulphur dosimetry pellets were enclosed in the sample containment vessel to determine the neutron fluence. From appropriate calibration curves, the total neutron fluence of neutrons with energies greater than 1 kev was determined as 4.2×10^{14} neutrons/cm.[2]

The kinetics of the vaporization of loosely-packed AP was determined by isothermal thermogravimetry with a commercial thermogravimetric analyzer (duPont model 951). A previously discussed procedure was employed in order to have the AP sample reach thermal equilibrium within 30 sec after the AP was inserted into the furnace.[8] The kinetic runs were carried out near 570K after it was discovered that the irradiated AP deflagrated at 590K. At 570K both the low-temperature decomposition and the vaporization of AP could be distinguished, and only weight loss corresponding to the vaporization of AP was used to determine the kinetics of the vaporization. Since the rate of vaporization of AP depends on sample size,[14,15] care was taken to have similar sample sizes